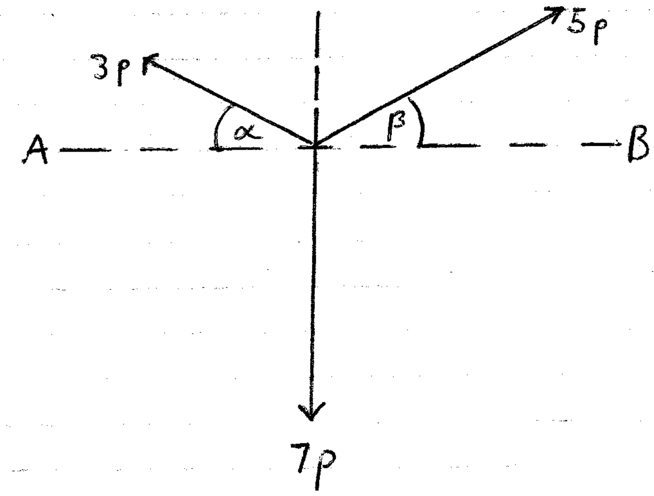
Mechanics Examples Sheet 1 – Solutions

1. Resolve parallel ($\uparrow\uparrow$) and perpendicular ($\perp$) to the line AB.



$\uparrow\uparrow$: $3\cos\alpha = 5\cos\beta$

$\perp$: $3\sin\alpha + 5\sin\beta = 7$.

On eliminating α:

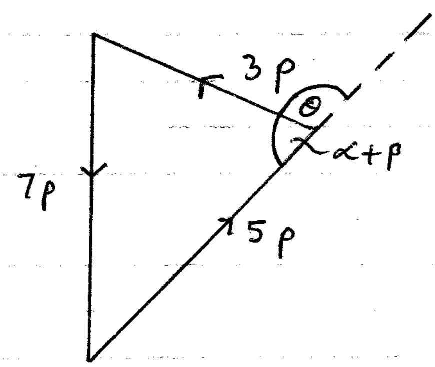
$$9 = (7 - 5\sin\beta)^2 + 25\cos^2\beta$$

$\Rightarrow \beta = \sin^{-1}\frac{13}{14}$ and, therefore, $\alpha = \sin^{-1}\frac{4}{14}$.

So, angle between $3p$ and $5p$ is

$$\pi - \sin^{-1}\frac{11}{14} - \sin^{-1}\frac{13}{14} = \underline{\underline{\cos^{-1}\frac{1}{2}}}$$

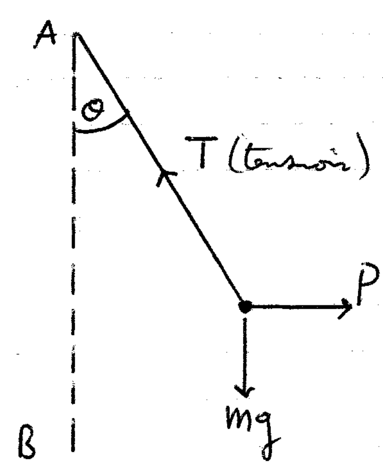
Alternatively, form a triangle of forces and use the cosine rule:



$$\cos(\alpha+\beta) = \frac{3^2 + 5^2 - 7^2}{2\times3\times5}$$

$$\Rightarrow \theta = \pi - \cos^{-1}\left(-\frac{1}{2}\right) = \underline{\underline{\cos^{-1}\frac{1}{2}}}$$

2. Resolve $\uparrow\uparrow$ and $\perp$ to AB:



$\uparrow\uparrow$: $T\cos\theta = mg$

$\perp$: $T\sin\theta = P$.

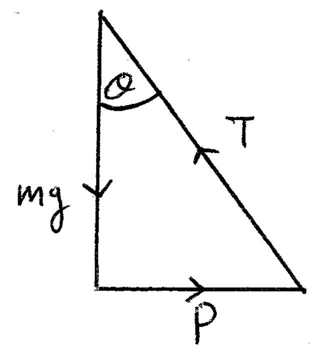
On dividing:

$$\underline{\underline{P = mg\tan\theta}}$$

Alternatively, form a triangle of forces:

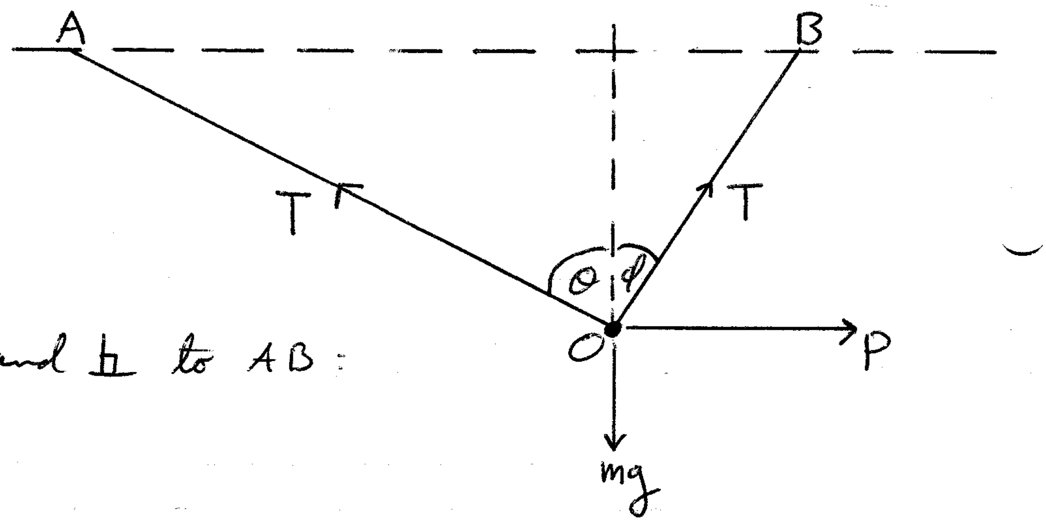
$$\sin\theta = \frac{P}{T}$$

$$\cos\theta = \frac{mg}{T}$$



$\Rightarrow \underline{\underline{P = mg\tan\theta}}$

3.



Resolve $\uparrow\uparrow$ and $\perp$ to AB:

($\uparrow\uparrow$) Horizontal forces on O : $T\sin\theta - T\sin\phi = P$ (1)

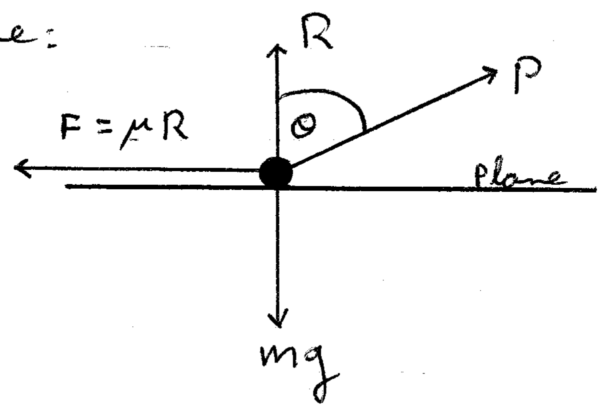
($\perp$) Vertical forces on O : $T\cos\theta + T\cos\phi = mg$ (2)

Note, from diagram, we have assumed that $\sin\theta > \sin\phi$. We could have assumed the opposite. Thus, on dividing (1) and (2) to eliminate T, the MAGNITUDE of P is the absolute value

$$\underline{\underline{P = \frac{|\sin\theta - \sin\phi|}{\cos\theta + \cos\phi}\, mg}}$$

4.

Resolve $\uparrow\uparrow$ and $\perp$ to plane:

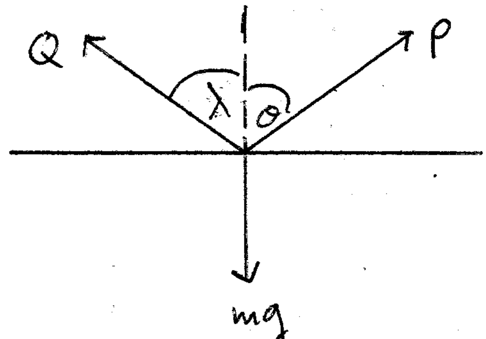


($\uparrow\uparrow$): $P\sin\theta = \mu R$

($\perp$): $P\cos\theta + R = mg$.

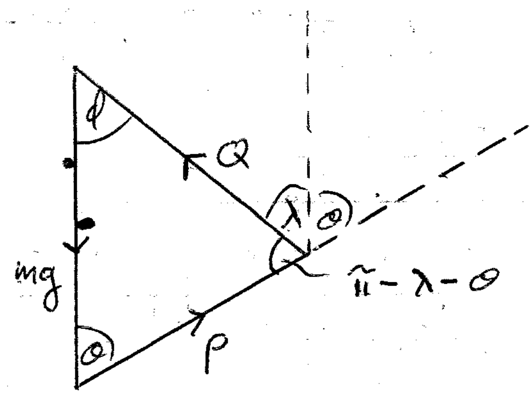
combine to eliminate R and the result follows.

Alternatively, form a triangle of forces by first combining the frictional force F and the normal reaction R :



where Q is the resultant force and $\tan\lambda = \frac{F}{R} = \mu$.

Thence,

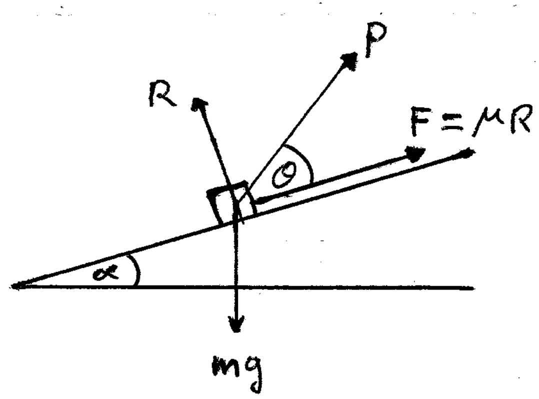


using the sine rule :

$$\frac{P}{\sin\phi} = \frac{mg}{\sin(\pi-\lambda-\theta)}.$$

The result follows with the aid of double angle formulae.

5.



Because the particle is on the point of slipping down the plane, the frictional force, F, acts up the plane. We will assume, for the purpose of drawing the diagram, that the force of magnitude P acts up the plane. Note that θ could be obtuse.

Let us resolve parallel and perpendicular to the plane.

$(\uparrow\uparrow)$: $P\cos\theta + \mu R = mg\sin\alpha$

$(\perp)$: $P\sin\theta + R = mg\cos\alpha$.

on eliminating R, we have

$$P(\cos\theta - \mu\sin\theta) = mg(\sin\alpha - \mu\cos\alpha)$$

or

$$P = mg\left(\frac{\sin\alpha - \tan\lambda\cos\alpha}{\cos\theta - \tan\lambda\sin\theta}\right)$$

on setting $\mu = \tan\lambda$. Factoring $(\cos\lambda)^{-1}$ and employing the appropriate double angle formulae yields

$$P = mg\left(\frac{\sin|\alpha - \lambda|}{\cos(\theta + \lambda)}\right).$$

If P is such that the particle is on the verge of slipping <u>up</u> the plane, the frictional force, F, will act <u>down</u> the plane. A similar calculation can be performed (as above). However, by allowing $\lambda \to -\lambda$ the required result is readily yielded:

$$P = mg\left(\frac{\sin(\alpha + \lambda)}{\cos(\theta - \lambda)}\right).$$

Note that the angle between the normal reaction, R, and the resultant, S, of the normal reaction and the frictional force, μR, is the <u>angle of friction</u>, λ: